

An abstract graphic featuring a cluster of colorful spheres (yellow, pink, and light green) on the left side, connected by white lines. The background is black with several glowing blue and yellow lines that create a sense of depth and movement, resembling a stylized molecular structure or a network diagram.

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## Aesthetical Beauty of Mathematics and the Pythagorean Theorem

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**Abstract.** We presented a brief discussion on the aesthetic beauty in its historical basis, referring to the philosopher and mathematician Pythagoras of Samos (c. 569 - c. 475 BC). The Pythagoreans have observed a clear connection between beauty and mathematics. One of the first mathematical knowledges of human history, the *Pythagorean Theorem*, considered the first main geometric information, has at least six qualities that can be attributed to mathematics in general: *universality*, *objectivity*, *truthfulness*, *aesthetics*, *resistance* and *applicability*. These attributes can also be credited to some degree to Arts. Just as great artists can achieve the goal of making their names universal from their illustrious masterpieces, the same occurs with those who studies and proposes mathematics. And what mathematics and art have in common? The answer is surprisingly positive: whatever type of art, whether painting, music, sculpture, dance, theater, film or poetry, art and math are based on abstraction, the use of imagination and primordial objects, as the shape or sound against numbers. In fact, mathematicians usually designate certain evidence by the adjective *elegant*, a very particular aesthetic term of this distinct area, and also extensively used for the characterization of artistic works.

### Introduction

Beauty don't wonder, is admired. Although humanity has always noticed, commented and contemplated beauty, the Greeks were the first to discuss and philosophize about its nature. Interestingly, they were also the same to appreciate and establish the knowledge we call mathematics, the universal and concise language of nature.

Beauty as the idea above all others (from the Greek *ιδέα*, or even *form*) was already commented by the Greek philosopher and mathematician Plato (c. 428 - c. 348 BC), disciple of Socrates (c. 470 - 399 BC). In his great dialogue *Phaedo* (or "On the Soul"), Plato deals with the last philosophical discussions of his master on his deathbed [1]. On discussing the immortality of the soul, he asked: "would not three be indestructible"?

In fact, the first explanations on the concept of beauty are due to pre-Socratic scholars such as the celebrated Greek philosopher and mathematician Pythagoras of Samos (c. 569 - c. 475 BC), see Fig. 1 [2]. The Pythagoreans already observed a clear connection between beauty and mathematics, for example when noticing the presence of the golden number<sup>1</sup> in nature, in the numerical relations of musical notes<sup>2</sup> or in an astonishing and illustrious theorem that is believed to be the first and most significant geometric notion ever.

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<sup>1</sup>The golden number (or even golden section, or ratio) was used to exhaustion in classical painting and architecture. Discovered by the Greeks, it has become an aesthetical beauty standard. In geometric terms it corresponds to a particular rectangle where the longer side (*a*), divided by the shorter (*b*), is equal to the division between the minor side and the difference between the longer and shorter sides. That is:  $a / b = b / (a-b)$ .

<sup>2</sup>The pentatonic scale: DO, RE, MI, SOL, LA, is another discovery attributed to Pythagoras, probably the first person to perform an experiment and to propose a theoretical basis. If a string generating certain note is divided in half, it would produce the same note but an octave above; Or when considering the same string divided into 3 parts, it generates another harmonic interval, and so on. In other words, specific proportions of different lengths, weights, or even volumes in musical instruments would provide harmonious sounds.



Figure 1. Detail of the Beautiful *Fresco* “The School of Athens” (1509 - 1510), by the Italian Renaissance Painter and Architect Raffaello Sanzio (1483 - 1520), Showing the Master Pythagoras Teaching Geometry and Music, Doing Notes in a Book. Vatican Museum, Stanza della Segnatura. Public domain.

But if mathematics is so beautiful, why many cannot see this? Why do children and young people be not fascinated, admired, enthused, enchanted, as they usually feel when listening to a song, or watching a painting, or even a theater / novel? The Pythagorean Theorem [3], states that “the square of the hypotenuse is equal to the sum of the squares of the other two sides”, as shown in Fig. 2. There are 370 ways to demonstrate this theorem [4].

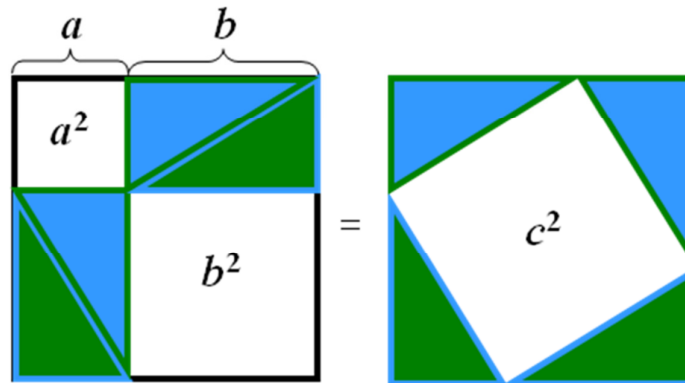


Figure 2. One of the 370 ways to demonstrate the Pythagorean theorem by simple rearrangement—just properly moving the triangles, the areas of squares of sides  $a$  and  $b$  show to be equivalent to the square of larger area (side  $c$ ). Considering two smaller squares, both of equal sides, such a figure resembles that found in a clay tablet of 1,800 BC, from the Babylonian collection of Yale Library, USA (YBC 7289)—that is, the singular theorem was already known by at least 1,300 years before Pythagoras. However, the understanding of this particular result as something unique and universal was only conceived by the Pythagoreans. Illustration by the author.

Although any of the 370 classic demonstrations on the proof of the Pythagorean Theorem would be considered, questions regarding learning difficulties remain. It is therefore proposed a relatively easy way to illustrate and introduce any mathematical subject in schools, by considering and discussing - through analogies, certain qualities of mathematics in general - and the Pythagorean Theorem in particular. For

example, this theorem could be compared so beautiful as a quote by the English playwright, actor and poet William Shakespeare (1564 - 1616): “To be, or not to be, that is the question” [5] (see Table 1). Although in art it is difficult to explain why a poetry, a song, a film or even a painting is beautiful, it is possible to highlight its details, the context and mainly the life and work of its artist.

Table 1. Excerpt from “The Tragedy of Hamlet, Prince of Denmark”, Act III, Scene I [5].

“To be, or not to be, that is the question:  
 Whether 'tis nobler in the mind to suffer  
 The slings and arrows of outrageous fortune,  
 Or to take arms against a sea of troubles,  
 And by opposing end them? To die, to sleep,  
 No more; and by a sleep to say we end  
 The heart-ache, and the thousand natural shocks  
 That flesh is heir to: 'tis a consummation  
 Devoutly to be wished. To die, to sleep;  
 To sleep, perchance to dream – ay, there's the rub:  
 For in that sleep of death what dreams may come,  
 When we have shuffled off this mortal coil,  
 Must give us pause – there's the respect  
 That makes calamity of so long life.  
 For who would bear the whips and scorns of time,  
 The oppressor's wrong, the proud man's contumely,  
 The pangs of despised love, the law's delay,  
 The insolence of office, and the spurns  
 That patient merit of the unworthy takes,  
 When he himself might his quietus make  
 With a bare bodkin? Who would fardels bear,  
 To grunt and sweat under a weary life,  
 But that the dread of something after death,  
 The undiscovered country from whose bourn  
 No traveller returns, puzzles the will,  
 And makes us rather bear those ills we have  
 Than fly to others that we know not of?  
 Thus conscience does make cowards of us all,  
 And thus the native hue of resolution  
 Is sicklied o'er with the pale cast of thought,  
 And enterprises of great pith and moment,  
 With this regard their currents turn awry,  
 And lose the name of action.”[...]

### **Some Qualities of Mathematics, and the Pythagorean Theorem in Particular**

From this analogy, the elaboration of the first great theorem of mathematics is due to someone, to an artist of the numbers and forms that we do not know for sure when he was born, even when and how he lived and died, who exactly was his ancestors and if he had descendants - practically what is known of Pythagoras is that he was one of the first great mathematicians [6]. His extraordinary figure is enveloped in mysticism, tales, legends, mysteries and anecdotes. It is undeniable its historical personality, even shrouded in mysteries. One of those who quoted him for knowing his works was Plato himself. It was probably Pythagoras who founded an order with rigid moral codes and with a remarkable characteristic—confidence in the study of mathematics as the basis for human conduct. Possibly he traveled through Egypt and Babylon, presumably having arrived in India, learning and absorbing knowledge on mathematics, music, astronomy, and philosophy [6].

Legend say that he was student of one of the seven wise men of antiquity, the Greek philosopher, astronomer, and engineer Thales of Miletus (c. 624 - c. 546 BC), probably the first great mathematician. On his return from his pilgrimages, he established himself on the southeastern coast of what is now Italy, founding an absolutely original, secret and communal order known as the "semicircle", where knowledge and property were considered common goods. For this reason, the discoveries were attributed to order, not to its members (who could be men or women, something rare in those times) [3]. Some of them called themselves *mathematikoi* (one of the meanings for "scientist" in Greek).

This order, or brotherhood, disseminated, among other things that certain symbols had mystical meaning and, at its deepest level, the reality of nature would be mathematical. They studied and proposed properties of numbers that should be known to any person: odd and even numbers, triangular and square numbers, natural or not (they are now called irrational), primes, among others... They assigned characteristics to numbers, as masculine and feminine, perfect, friendly ... They built at the same time the theoretical basis of numbers and geometry, until then a set of sparse and isolated rules.

In fact, there are at least six qualities of mathematics that are also observed in the Pythagorean theorem<sup>3</sup> (see also ref. [7]):

- i) *Universality*: the theorem does not deal to specific, but all triangles - the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. It is therefore a powerful statement, which can be understood by anyone, anywhere, at any time, in any language.
- ii) *Objectivity*: the theorem does not depend on interpretations, because it means the same for all who understand it. Of course to comprehend it, one has to get its idea, which can be concisely represented in a symbolic expression, the mathematical language:  $c^2 = a^2 + b^2$ , following Fig. 2.
- iii) *Veracity*: this idea is logical and consistent, and it is not possible to establish, for example, another result if the premises are the same. Therefore, it is impossible to derive a contradiction, nor redundancy.
- iv) *Aesthetics*: concise presentation of ideas resembles the correct choice as a color palette for a particular scene in a frame, or words in a poem or even notes in a song. The clear and logical arrangement of each of its arguments, demonstrations and proofs results in the impression that it is sufficient, unique and satisfactory - there is nothing to add, describe or even improve.
- v) *Resistance*: the Pythagorean theorem meant the same to the ancient Greeks as we did to us in our day, and there is reason to believe that it continues to mean the same to our descendants. This also occurs for all other theorems.
- vi) *Applicability*: although it is an old discussion involving the nature of mathematics, many professionals from this area still classify works as "pure" or "applied". At least it can be said that practically all the mathematics taught has some application, as the Pythagorean theorem. It is known, for example, that the ancient Egyptians long before Pythagoras used knotted ropes that, when stretched, represented a triangle with 3, 4, and 5 units each side (the unity could be anything, as the length of the forearm from the elbow to the tip of the middle finger, called cubit). The applicability of this particular triangle was to produce, for example, a 90 degree angle, *i.e.* a right angle - to quickly establish the contours of a squared area or even raise a wall. Rope was used because it was easier to carry [3].

It is important to emphasize that these qualities can also be applied to the Arts, as indeed they are for the *Queen and Servant of Sciences*, as mathematics was defined by Bell once [6].

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<sup>3</sup>There is a curiosity about the Pythagorean Theorem - as Pythagoras probably lived around the fifth century before Christ, there are records prior to his own discovery, about 3,800 years ago - as shown in the clay tablet called YBC 7289, from the Babylonian Collection of Yale Library, USA ([www.yale.edu/babylon](http://www.yale.edu/babylon)). The tablet was found in excavations of the ancient city of Nippur, circa 1899, the main scribal training center of the ancient Babylonian period. This particular tablet presents the earliest value of the  $\sqrt{2}$ , written in cuneiform characters. However, it is important to note that the understanding of this particular result, elevated to the condition of theorem, that is, as something universal, concise, true and unique was only conceived by the Pythagoreans.

## Many Aspects of the Pythagorean Theorem

When a mathematical statement is established, as a theorem, a proof is required. The Pythagorean theorem is unique in all mathematics to present various demonstrations of its essence and validity. An American mathematician, professor, engineer, and writer, Elisha Scott Loomis (1852 - 1940), collected, classified, discussed and published in 1927 an illustrated book with 370 different proofs of the Pythagorean Theorem [4]. In the first figure that illustrates this book the author wrote in bold letters: “Behold!” and then the Latin phrase: “*Viam Inveniam avt Faciam*”.

Such a phrase: “I shall either find a way or make one,” is credited to the Carthaginian general Hannibal (248 - c. 182 BC) in response to his commanders who had declared that would be impossible to cross the Alps with elephants during the Punic Wars, one of the longest in history (264 - 146 BC). Loomis’ message is clear: when there is no way to explain a demonstration or proof in mathematics, it is always important to keep in mind Hannibal’s quote: to seek a way, or to make one. In mathematics, Loomis showed that there are at least nearly four hundred paths to behold the veracity of the splendid Pythagorean theorem.

Still on the nature of mathematics, and following Loomis’ advice, any theorem requires *demonstration* (from the Latin “*demonstro*”, to show) or even *proof* (from Latin “*probo*”, credible, honest, virtuous). The first word demands a rationale based on one or more propositions that should result into certainty. The second requires an argument from experience (*i.e.*, prior knowledge) that leaves no doubts or uncertainties. A “theorem” can simply mean something as “I ask you to prove.” Loomis made an incredible collection of proofs of the Pythagorean theorem. Much later, in the 1991 edition of Guinness Book of World Records [8], Loomis’ work received an honorable mention related to “the theorem with the greatest number of proofs.”

Particularly on the aesthetic qualities of mathematics, the English philosopher, mathematician, logician, historian, critic and political activist Bertrand Arthur William Russell (1872 - 1970), Nobel Prize for Literature in 1950, stated in 1917: “Mathematics, rightly viewed, possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as in poetry [9].“

## Conclusions

Of course, great artists can achieve the goal of making their names universal from their illustrious achievements. Curiously, there is another quite specific and relatively old profession that also perpetuates its knowledge and discoveries.

This is the profession of the *mathematician*. It consists of a human task very different from others because it is always looking for great, unique and universal achievements. It is not every day that a discovery is done, whether it proves or demonstraes a reasoning that is generic, concise, objective, consistent, true, logical, permanent and independent of interpretations. If applicable, it’s much better, but not necessarily to be true *a priori*.

One might ask: what, if anything, do mathematics and art have in common? The answer is surprisingly positive. Regardless of the type of art, whether painting, music, sculpture, dance, theater, film or poetry, art and mathematics use abstraction as well as primordial objects, either form or sound against the number.

Otherwise, let’s see: both employ common terms, such as aesthetics, organization, perfection, and rigor. They seek harmony, balance and simplicity. They are considered *languages*. And one can observe that they search for patterns - interestingly, in both cases even an absence of patterns would be celebrated.

From immemorial times, primitive men and women manifested their curiosity and creativity through art, being dazzled by being the nature constituted by patterns and geometric forms. The evolution of painting shows a clear connection with the development of mathematical ideas.



Mathematics is thus, astonishingly simple, as is the Pythagorean Theorem - which has lasted forever. This is another lesson of the wise Greek master: to perpetuate a name it is enough to establish a new theorem, nothing different from what occurs in artistic interventions.

Just as in art, abstractions in mathematics are much celebrated. By the way, mathematicians usually designate certain proofs by *elegant*, an aesthetic term of this particular area. Of all sorts it is undeniable, practically a miracle that a mathematical knowledge, whether the Pythagorean Theorem or another, continues to exist, is understandable, true, universal, objective, accurate, consistent, and resistant to time, space, and interpretations. Probably mathematical concepts exist in a world apart, as conceived by the ancient Greeks since Pythagoras and Plato: a world of “ideas” or “forms” - where the beauty prevails, eternal and immutable.

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